Fredholmness of band-dominated Operators:
Limit Operators and Collective Compactness

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We study Fredholmness of a class of bounded and linear operators on a space $E = \ell^p(\mathbb{Z}^N, X)$, where $p \in [1, \infty]$, $N \in \mathbb{N}$ and $X$ is a complex Banach space. An operator $A$ on $E$ can be naturally identified with an infinite matrix $(a_{i,j})$ with indices $i,j \in \mathbb{Z}^N$ and entries in $L(X)$. We say that $A \in L(E)$ is band-dominated if it is the norm-limit of a sequence of operators with banded matrices.

For the study of Fredholmness of $A$, there has been extensive work by V. Rabinovich, S. Roch and B. Silbermann over the last decade [3]. The question of Fredholmness of $A$ is replaced by the invertibility of a family of local representatives – so-called limit operators – of $A$. The focus of attention has been widened to include the cases $p = 1, \infty$ by the second author in [2]. In this talk we use ideas of collective compactness to show that, for a big class of band-dominated operators on $\ell^\infty(\mathbb{Z}^N, X)$, the injectivity of all and surjectivity of some limit operators of $A$ are already sufficient for its Fredholmness. We demonstrate our results for the example of discrete Schrödinger operators and derive formulas for their essential spectra.

References

