Dynamic modelling of bank profits

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A topical issue in financial economics is the development of a stochastic dynamic model for bank behaviour. Under the assumption that the loan market is imperfectly competitive, we investigate the evolution of banking items such as loans, provisions for loan losses and deposit withdrawals, Treasuries and deposits and their relationship with profit. A motivation for studying this type of problem is the need to generalize the more traditional discrete-time models that are being used in the majority of studies that analyse banks and their operational idiosyncracies. An important outcome of our research is an explicit model for bank profit based solely on the stochastic dynamics of bank assets (loans, Treasuries and reserves) and liabilities (deposits). By way of conclusion, we provide a brief discussion of some of the economic aspects of the dynamic bank modelling undertaken in the main body of the article.

I. Introduction

An important problem in financial economics is to develop a workable dynamic model for bank profit by means of stochastic analysis. A motivation for studying this problem is to extend the discrete-time models used in the analysis of banking behaviour and regulation (see, for instance, Altug and Labadie, 1994 and Chami and Cosimano, 2001) to a more general class of models. Despite the extent of the existing literature on these issues, the use of dynamic bank models beyond two-periods is limited. Another motivating factor is that a broader stochastic calculus can potentially make the dynamic models tractable and widen the scope for risk analysis and regulation via the new framework provided by Basel II (see, for instance, Basel Committee on Banking Supervision, 2001, 2004 and Mukuddem-Petersen and Petersen, 2005).

A popular approach to the study of bank behaviour involves a loan market that is assumed to be imperfectly competitive. As a consequence, profits are ensured by virtue of the fact that the net loan interest margin is greater than the marginal resource cost of deposits and loans. Under this assumption, we present stochastic models of bank assets (loans and Treasuries) and liabilities (deposits) that, in turn, are used to describe bank profitability. As far as the literature on profitability, from the current journal, is concerned we highlight the paper Halkos and Georgiou (2005) (see, also, Granero and Reboredo (2005)). Here, it is demonstrated by means of a technical argument that the bank’s profits will not decrease if the growth rate of sales is higher than the absolute growth rate of the bank’s lending rate. The mathematical discussion contained in the article provides a condition for a bank

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remaining profitable. Their main assertions are supported by an econometric analysis that utilizes panel data from the Western European banking sector.

In our contribution, by contrast to Modigliani and Miller (1958), in the presence of competition imbalances, the value of the bank is dependent on its financial structure (see, for instance, Rochet, 1992). Several discussions related to bank modelling problems in discrete- and continuous-time settings have recently surfaced in the literature (see, for instance, Altug and Labadie, 1994; Chami and Cosimano, 2001; Repullo, 2004; Gheno, 2007). To a certain extent, the banking model that we derive is an analogue of the one presented in Altug and Labadie, 1994 (see, also, Chami and Cosimano, 2001). In particular, the latter mentioned contribution analyses the effect of monetary policy in an economy underpinned by banks operating in an imperfectly competitive loan market. In our article, a present value formula for continuous cash flows with continuous discounting plays an important role. In this regard, Gheno (2007) comments on traditional discounted cash flow models and their relation with the option value embedded in banks. Furthermore, in Rupello (2004), a discrete-time dynamic bank model of imperfect competition is presented.

The problem that we solve in this article may be stated as follows.

**Problem 1.1 (Dynamic modelling of bank profits):** Can we construct a stochastic dynamic model to describe the evolution of bank profit that considers provisioning for loan losses? (Section II).

II. Stochastic Banking Model

The Basel II capital accord (see, for instance, Basel Committee on Banking Supervision, 2001, 2004) permits internal models, that satisfy certain criteria, to be used by banks to determine the riskiness of their portfolios and the required capital cushion. In this spirit, we construct stochastic dynamic models of bank assets, $A_t$, and liabilities, $\Gamma_t$, that can specifically be identified as

$$A_t = \Lambda_t + T_t + R_t; \quad \Gamma_t = \Delta_t$$

where $\Lambda$, $T$, $R$ and $\Delta$ are loans, Treasuries, reserves and deposits, respectively. For sake of argument, in the sequel, we classify the sum of loans and reserves as a risky asset while Treasuries are considered to be riskless.

**Assets**

In this sub-section, the bank assets that we discuss are loans, provisions for loan losses, Treasuries, reserves and the total risky assets.

**Loans.** As a rule of thumb, the loan value that appears on the balance sheet can be equated with the bank’s investment (i.e. the *amount outstanding* or the *face value*) less a provision for bad and defaulting debts. This need for provisioning arises, since loans are initially not recorded at market value. In turn, this results from the fact that imputed market values are either empirically difficult to obtain due to the absence of traded markets or they rely on idiosyncratic assumptions. If assets were to be valued at market prices, as a first approximation, all changes in value would be unexpected, and banks would not be required to provision for potential losses.

In this paragraph, we provide a brief discussion of the current loan value. The current value of a loan at time $t$ denoted by $\Lambda_t$ can be defined as the discounted present value of the expected future cash flows generated by the loan. This definition leads to the formula

$$\Lambda_t = E \left[ \int_t^T \exp(-\delta(s-t))C_r \, ds \right]$$

(1)

where $T$ is the time of loan maturity, $C_r$ is the cash flows generated by the loan on the interval $[t, T]$ and $\delta$ is the discount rate. Under the assumption that the operating costs are zero, the expected cash flow is determined by subtracting the expected value of losses from default on the repayment of the contractual amounts from the contractual interest and principal payments on the loans. Furthermore, if $F$ is the face value of the loan and the loan rate, $r^A$, is equal to the discount rate, $\delta$, plus a default premium to compensate the bank for the probability that the borrower will not repay the loan, we can rewrite the current value of the loan as

$$\Lambda_t = F_t + E \left[ \int_t^T \exp(-\delta(s-t))D_r \, ds \right]$$

$$- E \left[ \int_t^T \exp(-\delta(s-t))l_r \, ds \right]$$

(2)

where $D$ is the default premium and $l$ is the expected loss from default on the repayment of the contractual amounts.
Provisions for loan losses. The bank’s investment in loans may yield substantial returns but may also result in loan losses. In line with reality, our dynamic banking model allows for loan losses for which provision can be made. The accompanying default risk is modelled as a compound Poisson process where $N$ is a Poisson process with a deterministic frequency parameter $\phi$. In this case, if the provisions for loan losses are defined so that they equal the difference between the face and current values of the loan, Equation 2 can easily be rearranged to demonstrate that provisions equal the difference between the discounted present value of expected losses, $l$, and the discounted present value of the expected default premiums, $D$. This description leads to the formula

$$P_t(l) = F_t(l) - A_t(l) = E \left[ \int_t^T \exp(-\delta(s-t))l_s \, ds \right]$$

$$- E \left[ \int_t^T \exp(-\delta(s-t))D_s \, ds \right]$$

Furthermore, we assume that the provisioning premium made by the bank for loan losses takes the form of a continuous contribution that can be expressed as

$$[1 + \theta(s)]\phi(s)E[P_t(L)]$$

where $\theta$ is a credit risk compensatory term, $\theta(t) \geq 0$ and $P_t$ is the actual provision for loan losses.

Treasuries. Treasuries, $T$, are bonds issued by national treasuries and are the debt financing instruments of the federal government. There are four types of Treasuries: Treasury bills, Treasury notes, Treasury bonds and savings bonds. All of the Treasuries besides savings bonds are very liquid and are heavily traded on the secondary market. We denote the interest rate on Treasuries or Treasury rate by $r_t^T$.

Reserves. Bank reserves are the deposits held in accounts with a national agency (for instance, the Federal Reserve for banks) plus money that is physically held by banks (vault cash). Such reserves are constituted by money that is not lent out but is earmarked to cater for withdrawals by depositors. Since, it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits will be needed as reserves. As a result of this description, we may introduce a reserve-deposit ratio, $\gamma$, for which

$$R_t = \gamma \Delta_t$$

The bank uses the remaining deposits to earn profit, either by issuing loans or by investing in assets such as Treasuries and stocks.

Total risky assets. We assume that the total risky asset process, $a$, follows the geometric Lévy process

$$da_t = a_t(\mu dt + \sigma dL_t)$$

where $\mu$ and $\sigma$ denotes the change and volatility in the total risky asset process and $L_t$ is a Lévy process with respect to a filtration, $(\mathcal{F}_t)_{t \geq 0}$, of the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$.

Liabilities

The bank takes deposits, $\Delta_t$, at a constant marginal cost, $c^\Delta$, that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the interest rate on deposits or deposit rate, $r_t^D$, is less than the Treasury rate, $r_t^T$. In the sequel, we express the sum of the marginal cost and deposit rate components, $k$, as

$$k_t = [r_t^D + c^\Delta] \Delta_t$$

The term $k$ in Equation 5 represents the consumption of the bank’s wealth by the deposit-taking activities of the bank. Hence, we refer to $k$ as the depository value.

We have to consider the possibility that unanticipated deposit withdrawals, $u$, will occur. By way of making provision for these withdrawals, the bank is inclined to hold Treasuries that are very liquid. In our contribution, we assume that $u$ is related to the probability density function, $f(u)$, that is, independent of time. For sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[\Delta, \Delta]$ so that the cost of liquidation, $c^u$, or additional external funding is a quadratic function of Treasuries and reserves. In addition, for any $t$ and $W = T + R$, if we have that

$$u > W_t$$

then bank assets are liquidated at some penalty rate, $r_t^P$. In this case, the cost of deposit withdrawals is

$$c^u(W_t) = r_t^P \int_{W_t}^\infty [u - W]f(u) \, du = \frac{r_t^P}{2\Delta} [\Delta - W_t]^2$$

Profit

Suppose that the value of the bank’s investment in risky assets, $a$, the depository value, $k$ and cost of
withdrawals, \( c^n(W_s) \), are given by Equations 4, 5 and 6, respectively. An expression for the dynamics of profits that may be deduced from the above is of the form

\[
d\Pi_t = \left[ r^T(s)\Pi_t + (\mu(s) - r^T(s))a_t + \mu^a(s) - k_s \right] dt - [1 + \theta(s)]P(l)\Pi_t dW_t - \left\{ 1 - P(l) \right\} dN_s, \quad s \geq t, \quad \Pi_t = \pi
\]

where \( \mu^a(s) \) is the rate term for auxiliary profits and \( N_s \) is a Poisson process.

**III. Discussion of the Economic Issues**

In the sequel, we briefly discuss some issues arising from the models for bank assets, liabilities and profits presented in Section II.

**Assets**

From Equation 2, we note that the current value of the loan, \( A \), will only equal its face value, \( F \), if the discounted present value of the expected default premiums, \( D \), is equal to the discounted present value of the expected losses, \( L \). In all likelihood, this condition will only be satisfied at the point at which the loan is granted. However, this may not be possible if decisions about pricing are distorted by competition or other factors. Importantly, the fact that \( E[l] \) in Equation 2 is positive does not necessarily imply that the bank expects to incur an overall loss on the loan. It is clear that the default premium also needs to be taken into account in this case. The procedure for modelling credit activity may be extended to take cognisance of the effect of macro-economic activity on the loan process (see, for instance, Altug and Labadie, 1994; Chami and Cosimano, 2001; Hackbarth et al., 2006).

In particular, there is considerable evidence to suggest that macro-economic conditions impact the probability of default and loss given default on loans (see, for instance, Hackbarth et al., 2006). The formula for provisions for loan losses given by Equation 3 raises a couple of important issues. First, aforementioned formulation establishes the relevance of entire future profile of expected losses and default premiums. This phenomenon is independent of the bank’s ability to liquidate or sell assets over a particular time horizon and is due to the fact that the current loan price will be affected by expected losses at any future time, regardless of the period over which the bank is able to terminate its exposure. Also, there is no need to make provision for loan losses if the default premium completely compensates the bank for the expected loss of principal and interest. Of course, in this situation, the expected losses from the default on repayments of the contractual loan amount will be exactly offset by the default premiums. If things happen according to plan, the bank will not experience an overall loss. Losses worse than expected, are by definition unexpected and would therefore, need to be covered by the bank’s capital. This point highlights the fact that provisions are needed only to cover expected credit losses in addition to those covered by the default premium incorporated in the loan rate.

**Liabilities**

In some contributions, it is standard to assume that the dynamics of the deposit rate process, \( r^\lambda = \{ r^\lambda_t \}_{t \geq 0} \), described in Sub-section Liabilities, is determined by the geometric Brownian motion process

\[
dr_t^\lambda = r_t^\lambda \left\{ \mu^\lambda dt + \sigma^\lambda dW_t^\lambda \right\}
\]

where \( \mu^\lambda \) and \( \sigma^\lambda \) are the drift coefficient and volatility in the deposit rate, respectively. The effects of the deposit rate being stochastic rather than constant can have a profound effect on the economic analysis of bank behaviour. For instance, in some quarters, \( r^\lambda \) is considered to be a strong approximation of banking monetary policy. Since, such policy is usually affected by macro-economic activity, \( M \), we expect \( r^\lambda \) and \( M \) to share an intimate connection. This interesting relationship is the subject of further investigation.

**Profits**

In Sub-section ‘Profit’ in Section II, the rate term for auxiliary profits, \( \mu^a(s) \), may be generated from activities such as special screening, monitoring, liquidity provision and access to the payment system. Also, this additional profit could arise from imperfect competition, barriers to entry, exclusive access to cheap deposits or tax benefits. The stochastic model for profit in Equation 7 can be considered to be the natural analogue of the corresponding discrete-time model presented in Altug and Labadie (1994) (see, also, Chami and Cosimano, 2001). In these contributions an alternative expression for profit in terms of dividends and bank capital
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(retained earnings and subordinate debt) is also discussed.

The model presented in Equation 7 lends itself to the formulation of a profit maximization problem (compare the contrasting discussion in Granero and Reboredo, 2005) that may involve optimal choices of the depository value, investment in risky assets and provisions for loan losses. In this regard, a realistic goal would be to maximize the expected utility of the discounted depository value during a fixed time interval, \([t, T]\), and final profit at time \(T\). The associated value function for this problem is given by

\[
V(\pi, t) = \sup_{\{k_s, a_s, P_i\}} E \left[ \int_t^T \exp[-\epsilon(s-t)] U^{(1)}(k_s) ds + \exp[-\epsilon(T-t)] U^{(2)}(\Pi_f) | \Pi_t = \pi \right]
\]

where \(U^{(1)}\) and \(U^{(2)}\) are utility functions and \(\epsilon > 0\) is the rate at which the depository value and terminal profit are discounted. In addition, we could place some restricting conditions on the optimization problem mentioned earlier by introducing constraints arising from cash flow, loan demand, financing and the balance sheet. Another factor that could influence the profit optimization procedure is banking regulation and supervision via the Basel II Capital Accord (see, for instance, Basel Committee on Banking Supervision, 2001, 2004). The aforementioned issues provide ample opportunities for future research.

References


