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The influence of language proficiency on geometric thinking

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Abstract:
Concern regarding the extent of underachievement in mathematics has been expressed by many authors. The role of language proficiency of the learner as a causal factor in this underachievement has been neglected. Researchers found sufficient evidence to conclude that language proficiency is related to mathematics achievement. In mathematics, symbolic language fills a dual role: as an instrument of communication and as an instrument of thought by making it possible to represent mathematical concepts, structures and relationships. According to Van Hiele language structure is a critical factor in the progression through the Van Hiele levels, from visual, concrete structures to abstract structures. This article reports on a research project that investigated the influence of language proficiency on geometric thinking. One hundred and fifty-two Grade 8 and 9 learners completed two tests each. One test measured language proficiency in their mother tongue and the second was a geometric test based on the Mayberry-type Van Hiele test for assessing learners' geometric thinking levels. Language proficiency was taken as the independent variable and geometric thinking as the dependent variable. In the analysis of the results, the top 25% and bottom 25% performers in the language proficiency test were taken. The performance of these two groups was compared to their performance in the geometry test, to determine if there was a significant difference in the performance of the more proficient language learners and the less proficient language learners with respect to each of the first three Van Hiele levels. Results showed a practical significant difference between the performance of the more proficient language learners and the less proficient language learners with respect to each of the first three Van Hiele levels, but also with respect to the geometry test as a whole. In particular, two aspects of language proficiency, namely, reading comprehension and vocabulary, appeared to be very strong predictors for geometric thinking on the first three Van Hiele levels.

Introduction
Concern regarding the extent of underachievement in mathematics (in South Africa, but also worldwide) has been expressed by many authors. According to research conducted by the Human Sciences Research Council (HSRC), South African learners in Grade 8, who took part in the Third International Mathematics and Science Study Repeat (TIMSS-R), ended up at the bottom of the list of 38 participating countries (Howie, 1999:9). National and international surveys of mathematical performance reveal that secondary school learners cannot identify and name shapes like the kite, rhombus, trapezium, parallelogram and triangle (Triadafillidis, 1995:225). Learners' performance is even poorer when it comes to items involving the understanding of features and properties of shapes.

Problem statement
The role of language proficiency as a causal factor in this underachievement has often been neglected. In this research project I attempted to find an answer to the following question: Does language proficiency have an influence on geometric thinking?

The role of language in the teaching and learning of mathematics

For a very long time the myth existed that mathematics is a language-free subject. Recent reform documents such as the National Council for Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards (1989:5) articulate communication as one of the five general goals for acquiring mathematical literacy. The South African Revised National Curriculum Statement (South Africa, 2002:1) states as one of its seven critical outcomes that learners should communicate effectively using visual, symbolic and/or language skills in various modes. These goals imply that learners should be exposed to various interrelated experiences that encourage them, amongst other things, to read, write and discuss mathematics.

The question of whether language proficiency is related to learning ability and academic achievement has been debated for many years. According to Secada (1992:637) there seems to be some relationship between degree of language proficiency in a given language and mathematics achievement in that language. He reported a study that included items for which bilingual respondents judged their fluency in their respective languages. For bilingual seniors from the 1980 cohort — whites and Hispanics — there was a significant relationship between their proficiency in English and their achievement in mathematics. In the studies he reviewed, Secada found sufficient evidence to conclude that language proficiency, no matter how it is measured, is related to mathematics achievement (Secada, 1992:639).

In mathematics, symbolic language, in both the teaching and learning of geometry, fills a dual role as an instrument of communication and as an instrument of thought by making it possible to represent mathematical concepts, structures and relationships (Esty & Teppo, 1996: 45).

Much of Vygotsky's account of cognitive development focuses on the role of language. He saw it as one of the most important psychological tools, and one of the most culturally developed ways of behaving toward subjects which allow high level cognitive functioning:

> When a new word has been learned by the child, its development is barely starting: the word at first is a generalization of the most primitive type; as the child's intellect develops, it is replaced by generalizations of a higher and higher type — a process that leads in the end to the formation of true concepts (Vygotsky, 1962:83).

Choat (1974:69) emphasizes the close interdependence of language and conceptual development:

> Even if the learner interacts with the physical aspect of the learning situation, i.e. objects, the verbal element is necessary both as a means of communication and as an instrument of individual representation in the acquisition of mathematical knowledge, a new concept brings a new word. Devoid of the conception, a child will not understand the word; without the word he cannot as easily assimilate and accommodate the concept.

According to Murray, Olivier and Human (1993:73), a problem-centred learning approach to mathematics teaching, compatible with a constructivist view of learning, encourages students to construct their own knowledge. This approach is based on the view that construction of mathematical knowledge is firstly an individual and secondly a social activity. Ernest (1991) says:

i) The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction.

ii) Interpersonal social processes are required to turn an individual's subjective mathematical knowledge into accepted objective mathematical knowledge.

Freudenthal (1973:416) explains that definitions in mathematics are "links in deductive thinking", therefore learning the characteristic properties of a geometrical shape is essential because they can form the basis of higher levels of thinking and help in gaining a practical and intuitive grasp of the mathematics of space (Triadafillidis, 1995:225).

Mathematics as a language

Much has been written on whether mathematics is a language or is like a language. A consensus on this issue does not exist in the mathematics community. Usiskin (1996:231-242) claims that mathematics is:

- a written language;
- a spoken language (oral language): the spoken language is important for the understanding of mathematical concepts;
- a symbolic (pictorial) language: since mathematics also studies geometry, it also studies the property of pictures, and at times these pictures themselves constitute part of the language of mathematics.

Other researchers (e.g. Magnus, 1997:266) dispute the claim that mathematics is a language, preferring to describe it as making use of a special language, and having some functions of a language, but not being a language in its own right. Bullock (1994:736) disagrees, claiming that indeed mathematics is a new and separate language.

Pimm (1987:2) reconciles these apparently conflicting notions when he claims that mathematics is a language in the metaphorical sense. He refers to a "register" — a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. He believes that metaphor is central to the development of the mathematics register.

Although I do not regard mathematics as a language *per se*, I do agree that language plays an important role in the teaching of mathematics, and that mathematics makes use of a special language. Appropriate mathematical language should help learners gain conceptual understanding, rather than confuse them (Tracy, 1994:221). The real problem is teachers' lack of awareness of how powerful their mathematical language can be in a geometry classroom.

Recognizing the fact, that language plays an important role in mathematics, forces one to rethink its teaching.

The Van Hiele levels of geometrical thinking

In this study, the Van Hiele levels of thinking were used. This theory was developed by Dina van Hiele-Geldof and her husband Pierre-Marie van Hiele in the Netherlands. According to the Van Hieles, the learner, assisted by appropriate instructional experiences, passes through five levels, which are sometimes reduced to the following four levels (Van Hiele, 1999:311):

- Level 1 is the visual level, which begins with nonverbal thinking. The learner identifies, names, compares and operates on geometric figures according to their appearance: "It is a square because it looks like one".
- Level 2 is the descriptive level, where figures are recognised by their properties. At this level, language is important for describing shapes. However, at this level, properties are not yet logically ordered.
- Level 3 is the informal deduction level, where properties are logically ordered. Properties are deduced from one another. Students use properties that they already know to formulate definitions, for example, for squares, rectangles and equilateral triangles, and use these to justify relationships, such as explaining why all squares are rectangles. However, at this level, the intrinsic meaning of deduction, is not understood (Van Hiele, 1999:311). Van Hiele also contends that learners have often not achieved this level of informal deduction, and consequently are not successful in their study of Euclidean geometry.
- Level 4 is the formal deduction level, the learner proves theorems deductively and establishes interrelationships among networks of theorems.

Van Hiele (1999:311) concluded that the development of these levels is more dependent on instruction and learning than on age and maturation, and that the development from one level to the next should include sequences of activities, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learned into what they already know.

According to Van Hiele (Fuys *et al*., 1988:5-6), language structure is a critical factor in the movement through the levels, from the concrete structures to abstract structures. In stressing the importance of language, Van Hiele points out that many failures in teaching geometry result from a language barrier — the teacher using the language of a higher level than is understood by the learner.

Motivation for study

The value of this study lies in the fact that it can contribute to the improvement of geometry teaching and learning. It can also focus teachers' attention on the important interaction between language and geometrical thinking, This study further emphasized the importance of the correct use of subject terminology in the forming of the correct mathematical concepts.

Method of investigation

In this empirical study, a population of 152 Grade 8 and 9 learners were given two question papers each, one on language proficiency, and a geometric paper based on the Mayberry-type Van Hiele test (Lawrie, 1998) for measuring geometric thinking levels.

- The language proficiency test, developed from the ELSA test (Hough & Horne, s.a.) consists of six questions: (i) a comprehension test, (ii) a dictation, (iii) phonetics, (iv) a tagrukon, (v) clozure test and (vi) a question on vocabulary.
- The geometric test consists of 24 questions which are based on the first three Van Hiele levels. Five concepts on each level were included: the circle, square, right-angled triangle, isosceles triangle and parallel lines.

Test items

Geometry test

Figure 1 shows an example of a few geometry test items on each of the first three levels based on the square.

Level 1

5. Which of these figures are squares?

A	B	C	D	E	F

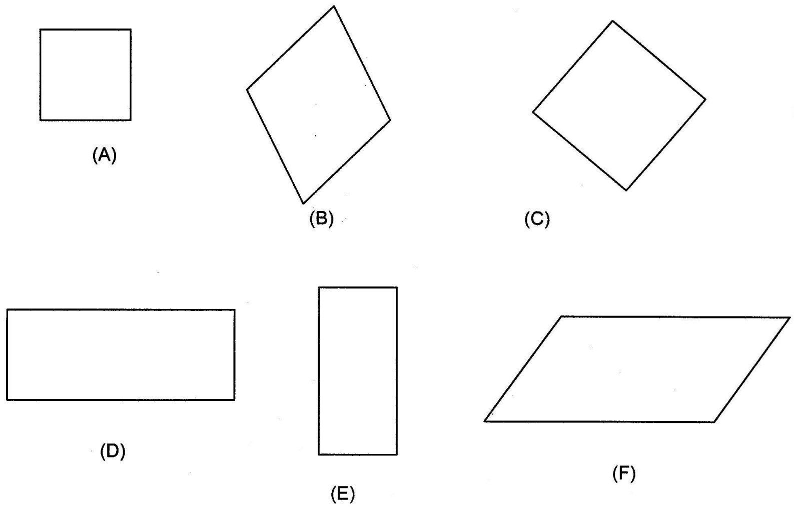

Figure 1

Level 2

11.1 Draw a square
11.2 What must be true about the sides?
11.3 What must be true about the angles?
16.1 Name some ways in which squares and rectangles are alike

Level 3

16.2 Are all squares also rectangles? YES / NO
16.3 Why?

Scoring system

The language proficiency test was administered by an accredited person. Each of the questions had to be answered within a certain time limit. The questions were then marked according to a preset memorandum.

The geometry test was marked according to the scale in Table 1 (Gutiérrez, Jaime & Fortuny, 1991:240).

Table 2 shows the numerical weight allocated to each type of answer (Gutiérrez *et al.*, 1991:241).

Statistical procedures and techniques

The data of the two tests were processed by using The SAS System for Windows Release 6.12 (SAS INSTITUTE, 1996). Effect sizes (Steyn, 1999:3) were used to determine if there were any significant differences between the learners' language proficiency and their geometric thinking.

Table 1

Type	Response
Type 0	No answer, or answer could not be categorized.
Type 1	Answers indicate that learners have not reached a given level
Type 2	Answers consist of incorrect explanation, reasoning or results.
Type 3	Correct, insufficient answer to indicate a given level. Answers contain few and rudimentary reasoning.
Type 4	Correct or incorrect answers. Characteristics of two consecutive levels. Clear and sufficient reasoning.
Type 5	Reasoning complete but incorrect or reasoning correct, but no solution.
Type 6	Answers are correct and reflect given level, but reasoning is incomplete.
Type 7	Answers are correct, complete and sufficiently justified and reflect given level of reasoning.

Table 2

Type	0	1	2	3	4	5	6	7
Weight	0	0	20	25	50	75	80	100

Language proficiency was the independent variable, and geometric thinking the dependent variable. Then the top and bottom 25% performers on the language proficiency test were taken, and their performance at the different Van Hiele levels compared.

In the statistical analysis, the standardised difference was used, i.e. the difference between the two means, of the above-mentioned groups, divided by the standard deviation. Cohen (1988: 20-27) calls this value *d*. This value was used to determine if there were any practically significant differences between the geometric thinking of the more successful learners and the less successful learners. Cohen uses the following scale for the *d* values:

d = 0.2: small effect, which indicates that the experiment should rather be repeated, or that the result is non-significant

d = 0.5: medium effect, which may refer to significance

d = 0.8: large effect, which indicates that the results are significant and of practical importance.

Discussion of results

The aim with this study was to determine whether there is a relationship between language proficiency and geometrical thinking. The results are summarised in Tables 3, 4, 5, and 6.

Table 3 Language proficiency in comparison with Van Hiele level 1

Language test	Successful learners (top 25%)		Less successful learners (bottom 25%)		*d* values
	Mean	Std deviation	Mean	Std deviation	
Question 1	75.6	13.1	56.7	15.3	1.2
Question 2	75.4	13.9	65.5	20.6	0.5
Question 3	66.2	19.6	47.9	24.0	0.8
Question 4	66.7	19.0	46.1	21.4	1.0
Question 5	65.9	15.7	49.5	17.9	0.9
Question 6	56.8	14.1	36.8	13.0	1.4
Total	65.2	9.1	46.6	11.8	1.5

For the relationship between language proficiency and the first of the Van Hiele levels (the visual level), in almost all of the questions, an effect size of $d \geq 0.8$ was obtained, indicating a practically significant difference between these two groups of students. It therefore seemed as if the more proficient language learners did better on the visual level than the less proficient language learners.

Table 4 Language proficiency in comparison with Van Hiele level 2

Language test	Successful learners (top 25%)		Less successful learners (bottom 25%)		*d* values
	Mean	Std deviation	Mean	Std deviation	
Question 1	78.2	10.1	58.4	16.3	1.2
Question 2	79.7	13.5	68.2	20.3	0.6
Question 3	65.8	18.5	51.1	26.6	0.6
Question 4	65.5	16.9	52.6	28.4	0.5
Question 5	70.5	12.3	53.2	15.4	1.1
Question 6	57.9	14.7	40.9	15.9	1.1
Total	70.2	7.2	53.9	14.7	1.1

For the relationship between language proficiency and the second Van Hiele level (the descriptive level), an effect size of $d \geq 0.8$ was obtained in questions 1, 5, 6, and the total, indicating a practically significant difference between these two groups of students with regard to reading comprehension, sentence closure, vocabulary, and the language test as a whole. In questions 2, 3, and 4 a medium effect size of $d \geq 0.5$ was obtained, indicating a medium effect in the difference between the performance of the more proficient language learners in the questions on phonetics, dictation and the tagrukon.

Table 5 Language proficiency in comparison with Van Hiele level 3

Language test	Successful learners (top 25%)		Less successful learners (bottom 25%)		*d* values
	Mean	Std deviation	Mean	Std deviation	
Question 1	72.9	15.4	59.6	15.6	0.9
Question 2	73.9	16.7	67.9	23.2	0.3
Question 3	64.2	20.9	55.8	28.0	0.3
Question 4	59.7	19.5	53.7	28.0	0.2
Question 5	62.4	14.2	55.3	17.5	0.4
Question 6	54.9	15.5	44.0	14.6	0.7
Total	65.2	10.6	55.7	14.8	0.6

With respect to the role of language proficiency on the third Van Hiele level (the informal deduction level), the following results were obtained: in question 1, an effect size of $d = 0.9$ was obtained, indicating a practically significant difference between these two groups of students with regard to reading comprehension. In question 6 and the total of the language test a medium effect size of $d \geq 0.5$ was obtained, indicating a difference of medium effect between the performance of the more proficient language learners in the question on vocabulary and the language test as a whole. In the remaining questions, the differences were not practically significant ($d < 0.5$). It therefore seems as if the more proficient language learners in the reading comprehension and the vocabulary did better in the informal deduction level where properties of geometric figures are logically ordered.

Table 6 Language proficiency in comparison with Van Hiele (total)

Language test	Successful learners (top 25%)		Less successful learners (bottom 25%)		*d* values
	Mean	Std deviation	Mean	Std deviation	
Question 1	79.3	10.3	58.5	17.4	1.2
Question 2	77.4	15.2	65.9	22.2	0.5
Question 3	70.0	18.5	49.7	25.9	0.8
Question 4	67.6	17.2	51.0	27.0	0.6
Question 5	69.2	14.0	50.5	18.8	1.0
Question 6	61.6	12.7	40.3	16.6	1.4
Total	71.5	7.2	52.8	15.4	1.2

From the results obtained it seems that the relationship between especially reading comprehension and general vocabulary (as independent variables) and geometrical thinking (as dependent variable) on the first three levels is practically significant, because in all the cases a large effect size was established.

Conclusions

From the results of this study it is clear that language is a determining factor in higher order thinking skills. The learning and teaching of mathematics is only possible through the use of language. The fact, that geometry concepts play an important role as a link between mathematics and language, implies that learners' inadequate language proficiency will inhibit their learning of geometry.

The conclusion can be drawn from this study that language proficiency has indeed an impact on geometrical thinking. This finding is consistent with Van Hiele's statement that language is important in the movement from one level to the next (Fuys *et al*., 1988:5).

Recommendations

1. Mathematics teachers will have to give attention to general use of language, as well as subject specific language in the geometry class. This includes correct use of language by the teacher, practising of geometric terminology, as well as monitoring of the use of language by the learners.
2. Attention should also be given to learners' reading comprehension, and this responsibility lies not only with the language teacher, but with every teacher involved in the teaching of the learners.
3. More opportunity must be given to learners to communicate geometrical concepts to one another during group work, since this is one way that learners can express their thoughts, and be able to conceptualize.

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