2 Algebra

Monomial modular representations and symmetric generation of groups

Robert T. Curtis

Department of Mathematics and Statistics
University of Birmingham, Edgbaston
Birmingham, England

SAMS Subject Classification: 2

A (complex) linear representation of a finite group is a homomorphism from the group into the multiplicative group of the complex numbers. If a linear representation of a subgroup $H$ of a group $N$ is induced up to $N$ we obtain a monomial representation of $N$ of degree $n$ equal to the index of $H$ in $N$, the non-zero entries of whose matrices are all $m$th roots of unity for some $m$. If the field of complex numbers is replaced by some prime field $\mathbb{Z}_p$ which possesses $m$th roots of unity, then these matrices can be interpreted as automorphisms of a free product of $n$ copies of the cyclic group of order $p$. We denote this free product by $p^n$, and form the progenitor $P = p^n : N$, a split extension of this infinite group by $N$. It turns out that many of the sporadic simple groups can be defined in a revealing way as homomorphic images of such progenitors.

This process is described in detail and various examples, including the Mathieu groups $M_{11}, M_{23}$, the Held group $He$ and the Harada-Norton group $HN$, are described.

On supernilpotent nonspecial radicals

H. France-Jackson

Department of Mathematics
University of Port Elizabeth (Vista Campus)

SAMS Subject Classification: 2

A radical $\varrho$ is called trivial if $\varrho$ coincides with the class of all associative rings. A hereditary radical $\alpha$ is called supernilpotent if $\alpha$ contains all the nilpotent rings. The upper radical $\mathcal{U}(\mu)$ generated by a hereditary and essentially closed class $\mu$ of prime rings is called a special radical. It is well known that a supernilpotent radical $\alpha$ is special if and only if $\alpha = \mathcal{U}(\pi(\alpha))$, where $\pi(\alpha)$ denotes the class of all prime and $\alpha$-semisimple rings.

In this talk, for every prime number $p$ we will construct a non-trivial supernilpotent radical $\alpha_p$ for which the class $\pi(\alpha_p)$ is empty which makes $\alpha_p$ nonspecial. The radicals $\alpha_p$ form a generalization of Ryabukhin’s example of a supernilpotent and a nonspecial radical [1].
References


Prime ideals in group near-rings

JH Meyer

Department of Mathematics & Applied Mathematics,
Free State University

SAMS Subject Classification: 2

The flow of primality (for various interpretations of primeness) between the base near-ring \( R \) and the group near-ring \( R[G] \) (where \( G \) is a multiplicatively written group) is discussed and illustrated by examples. Some of the illustrations will focus on a computer-generated example of a distributively generated group near-ring.

On the exact spread of finite simple groups

Jamshid Moori

School of Mathematical Sciences
University of KwaZulu-Natal, Pietermaritzburg

SAMS Subject Classification: 2

A group is 2-generated if it can be generated by two elements \( x \) and \( y \). In this case \( y \) is called a mate for \( x \). Brenner and Wiegold [3] defined a finite group \( G \) to have spread \( r \). A group is said to have exact spread \( r \) if it has spread \( r \) but not \( r + 1 \). The exact spread of a group \( G \) is denoted by \( s(G) \). M S Ganief [1] in his PhD thesis proved that if \( G \) is a sporadic simple group, then \( s(G) \geq 2 \). In [2] the author and Ganief used probabilistic methods and established reasonable lower bound for the exact spread \( s(G) \) for each of the sporadic simple group \( G \). The present paper deals with the search in finding reasonable upper bounds for the exact spread of the sporadic simple groups. The work is completed for thirteen of the sporadics, while for the remaining thirteen it is still in progress.

References

On sub-idempotent radicals and class pairs
W. A. Olivier

Department of Mathematics
University of Port Elizabeth

SAMS Subject Classification: 2

[No abstract was available at time of printing.]

On the stabilizers of the minimum-weight codewords of the dual binary codes from triangular graphs
B. G. Rodrigues

Departmento de Matemática e Engenharia Geográfica
Universidade Agostinho Neto
Luanda, Angola

SAMS Subject Classification: 2, 4, 14.

The stabilizers of the minimum-weight codewords of dual binary codes obtained from the strongly regular graphs $T(n)$ defined by the primitive rank-3 action of the alternating groups $A_n$ where $n \geq 5$, on $\Omega^{(2)}$, the set of duads of $\Omega = \{1, 2, \cdots, n\}$ are examined.

Algebraic aspects of the deduction theorem
C. J. van Alten

School of Mathematics
University of the Witwatersrand

SAMS Subject Classification: 2, 1

Given any algebra $A$ one may define an associated logic, say $L(A)$, whose relationship to $A$ is similar to that between the classical propositional calculus and the two-element Boolean algebra.

By a deduction theorem for a logic we mean a property of the type:

$$\Gamma \cup \{\psi\} \vdash \varphi \Rightarrow \Gamma \vdash \psi \rightarrow \varphi.$$
The following question arises: what properties are required of the algebra $A$ for the logic $L(A)$ to have a deduction theorem?

There exists a general result that $L(A)$ has a deduction theorem if and only if the variety generated by $A$ (i.e., $V(A)$) has equationally definable principle congruences (i.e., EDPC). That is, a strong congruence property is required for the whole class $V(A)$. For certain types of algebras $A$, I shall present a condition on $A$ alone that will ensure that $V(A)$ has EDPC and hence $L(A)$ has a deduction theorem.

Modules whose hereditary pretorsion classes are closed under products

John E. van den Berg*1 and Robert Wisbauer2

1School of Mathematical Sciences
University of KwaZulu-Natal
Pietermaritzburg

2Mathematics Institute
University of Düsseldorf
Germany

Institution SAMS Subject Classification: 2

A module $M$ is called product closed if every hereditary pretorsion class in $\sigma[M]$ is closed under products in $\sigma[M]$. Every module which is locally of finite length is product closed and every product closed module is semilocal. Let $M \in \text{Mod} R$ be product closed and projective in $\sigma[M]$. It is shown that (1) $M$ is semiartinian; (2) if $M$ is finitely generated then $M$ satisfies the DCC on fully invariant submodules; (3) $M$ has finite length if $M$ is finitely generated and every hereditary pretorsion class in $\sigma[M]$ is $M$-dominated. An example is provided which shows that (3) fails to hold if this latter condition is dispensed with. If the ring $R$ is commutative it is proven that $M$ is product closed if and only if $M$ is locally of finite length.
Can the non-diagonal tile in a generalized matrix ring be recovered?
Sorin Dascalescu and Leon van Wyk*

Department of Mathematics  
Stellenbosch University

SAMS Subject Classification: 2

In this talk we provide an example regarding the impossibility of the recovery up to isomorphism of one of the algebraic structures involved in generalized triangular matrix rings. To be more precise: we show that the tile in the non-diagonal position of a $2 \times 2$ upper triangular tiled matrix ring cannot be recovered up to isomorphism even if the base ring is finite. This ties in with recent work on the recovery/non-recovery of some of the algebraic structures involved in various classes of matrix rings.

---

An $S_n$-invariant subgroup of $\mathbb{Z}_m^n$
Kenneth Zimba

School of Computational and Mathematical Sciences  
University of the North  
Sovenga

SAMS Subject Classification: 2

Let $\mathbb{Z}_m$ be the cyclic group of order $m$ and $N = \mathbb{Z}_m^n$ be the direct product of $n$ copies of $\mathbb{Z}_m$. Let $S_n$ be the symmetric group of degree $n$. The wreath product $N \rtimes S_n$, which is a split extension of $N$ by $S_n$, is called the generalized symmetric group $B(m, n)$ (see [1]). In [2], an $S_6$-invariant submodule $2^5$ of $2^6$ is given in terms of the orbits of the action of $S_6$ on $2^6$. In this talk we give an $S_n$-invariant subgroup $S = \mathbb{Z}_m^{n-1}$ of $N$. Since the group $S$ is $S_n$-invariant, it forms a split extension $S:S_n$ with $S_n$. Such group extensions arise as subgroups of simple groups whose structures require study (for example $2^5:S_6$ is a maximal subgroup of $SP(2,6)$).

References

[3] J. Moori and K. Zimba, A permutation action of the Symmetric group $S_n$ on the groups $\mathbb{Z}_m^n$ and $\overline{\mathbb{Z}}_m^n$, submitted.