14 Combinatorics

The protection of complete multipartite graphs

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SAMS Subject Classification: 14

The notion of higher order domination in graphs has been studied in the literature and may be categorised as so–called *finite higher order domination* and *infinite higher order domination*. The former concept concerns the protection strategies for graphs against a *finite* number of attacks, while the latter caters for an *infinite* number of attacks. Although the infinite order domination parameters have been established for the complete multipartite graph, only the *secure* finite order parameters have been resolved, and only for the complete bipartite case. In this talk, both the *smart* and *foolproof* versions of the *secure* and *weak Roman* higher order domination parameters are fully established for the complete multipartite graph.

Average value and average position of weak large left-to-right maxima

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In this talk we shall consider strings of characters, or *words*, of length n, from a fixed alphabet. These letters are independently generated with a geometric probability in such a way that the letter i has probability pq^{i-1} where $p+q=1$.

An element is a left-to-right maximum if it is larger (in the strict sense) or larger or equal (in the weak sense) than all the elements to its left. We are counting the position r of these maxima from the right, so that for example the last left-to-right maximum has position $r=1$.

In this talk we will discuss, *in the weak sense*, the average value and average position of the rth left-to-right maximum counted from the right.

On the connectivity of diamond-free graphs

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SAMS Subject Classification: 14

Let G be a graph of order $n(G)$, minimum degree $\delta(G)$ and connectivity $\kappa(G)$. Chartrand and Harary gave the following lower bound on the connectivity

$$\kappa(G) \geq 2\delta(G) + 2 - n(G).$$

Topp and Volkmann improved this bound to $\kappa(G) \geq 4\delta(G) - n(G)$, if G is bipartite and $\kappa(G) < \delta(G)$. We show that this result remains valid for diamond-free graphs with $\delta(G) \geq 3$. A diamond is a graph obtained from a complete graph with 4 vertices by removing an arbitrary edge.
Furthermore, we show that the above bounds can be improved significantly for graphs with $\delta(G) \geq 3$ and no 4-cycle, namely, if $\kappa(G) < \delta(G)$, then

$$\kappa(G) \geq 2\delta(G)^2 + 2 - 2\delta(G) - n(G).$$

For graphs that, in addition, contain no 3-cycle, we improve this bound even further.
This is joint work with Angelika Hellwig and Lutz Volkmann (RWTH Aachen, Germany).

Broadcasts in graphs

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We say that a function $f : V \to \{0, 1, \ldots, diam(G)\}$ is a *broadcast* if for every vertex $v \in V$, $f(v) \leq e(v)$, where $diam(G)$ denotes the diameter of G and $e(v)$ denotes the eccentricity of v. The *cost* of a broadcast is the value $f(V) = \Sigma_{v \in V} f(v)$. We introduce and study the minimum and maximum costs of several types of broadcasts in graphs, including dominating, independent, and efficient broadcasts.

The depression of a graph — Part 2

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SAMS Subject Classification: 14

Unlike the case of altitude, there is no forbidden subgraph characterization of graphs with depression two, because if any vertex of an arbitrary graph is joined to two new vertices, the resulting graph has depression two. In fact, in this paper we show that a connected graph has depression two if and only if it has a vertex adjacent to two leaves or to two adjacent vertices of degree two.

A linear Vizing-like relation relating the size and total domination number of a graph

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SAMS Subject Classification: 14

We prove that $m \leq \Delta(n - \gamma_t)$ for every graph, each component of which has order at least 3, of order n, size m, total domination number γ_t and maximum degree $\Delta \geq 3$.

Stratification and domination in graphs

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SAMS Subject Classification: 14

A graph G is 2-stratified if its vertex set is partitioned into two classes (each of which is a stratum or a color class.) We color the vertices in one color class red and the other color class blue. Let F be a 2-stratified graph with one fixed blue vertex v specified. We say that F is rooted at v. The F-domination number of a graph G is the minimum number of red vertices of G in a red-blue coloring of the vertices of G such that every blue vertex v of G belongs to a copy of F rooted at v. In this paper we investigate the F-domination number when (i) F is a 2-stratified path P_3 on three vertices rooted at a blue vertex which is a vertex of degree 1 in the P_3 and is adjacent to a blue vertex and with the remaining vertex colored red, and (ii) F is a 2-stratified K_3 rooted at a blue vertex and with exactly one red vertex.

The radius of a graph

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SAMS Subject Classification: 14

Let G be a connected graph. The distance between two vertices u, v of G is the length of a shortest path joining u and v in G. The eccentricity of a vertex v of G is defined as the distance between v and a vertex farthest from v. The radius of the graph G is the minimum of the eccentricities of its vertices. In this talk, we present bounds on the radius of a graph.

Deviations of the generalized Petersen graphs

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SAMS Subject Classification: 14

For a positive integer $n \geq 3$, and $1 \leq r \leq \frac{n}{2}$, *the generalized Petersen graph* $G(n,r)$ has vertices $\{u_0, u_1, \cdots, u_{n-1}, v_0, v_1, \cdots, v_{n-1}\}$ and edges of the form $[u_i, u_{i+1}], [u_i, v_i], [v_i, v_{i+r}], i \in \{0, 1, \cdots, n-1\}$ with the arithmetic modulo n. Frucht, Graver and Watkins determined the automorphism group of $G(n,r)$ for each n and r.

In a classical result of Sabidussi, it was shown that every vertex-transitive graph G has a multiple kG which is Cayley. In this sense, therefore, the deviation of a vertex-transitive graph G was defined as the smallest k such that kG is Cayley. This parameter of vertex-transitive graphs, it is envisaged, would measure how far they are at being Cayley.

We shall discuss the deviations of vertex-transitive generalized Petersen graphs.

A Sylvester-Gallai theorem for quaternions

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SAMS Subject Classification: 14, 4

A Sylvester-Gallai (SG) configuration is a finite set S of points such that the line through any two points in S contains a third point of S. According to the Sylvester-Gallai Theorem, an SG configuration in real projective space must be collinear. A problem of Serre (1966) asks whether an SG configuration in a complex projective space must be coplanar. This was proved by Kelly (1986) using a deep inequality of Hirzebruch. We give an elementary proof of this result, and then extend it to show that an SG configuration in projective space over the quaternions must be contained in a three-dimensional flat.

Classifying trees labeled with a condition at distance two

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SAMS Subject Classification: 14

The problem of assigning channels to radio transmitters can be modeled in various ways. Griggs and Yeh modeled the problem by representing transmitters as the vertices of a graph; adjacent vertices receive channels at least two apart, and vertices that are distance two from each other receive distinct channels. For a given graph G, an $L(2,1)$-*labeling* is defined as a function $f : V(G) \longrightarrow \{0, 1, 2, ...\}$ such that $|f(u) - f(v)| \geq 2$ for all u, v where $uv \in E(G)$ and $|f(u) - f(v)| \geq 1$ for all u, v where $d(u, v) = 2$. The *span* of f, denoted by $span(f)$, is defined as $\max f(u)$ for all $u \in V(G)$. The $L(2,1)$-*number* of a graph G is denoted by $\lambda(G)$ and is defined as the minimum span over all $L(2,1)$-labelings of G. It has been shown that for any tree T of maximum degree Δ, $\Delta + 1 \leq \lambda(T) \leq \Delta + 2$. The aim of this paper is to provide a characterization of those trees which have an $L(2,1)$-number of $\Delta + 1$ and then study these trees when $\Delta = 1$.

The number of edges in a bipartite graph of given order and radius

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SAMS Subject Classification: 14

In a graph G, the *distance* between two vertices, u and v, is the length of a shortest path between them. The *eccentricity* of a vertex v is the distance between v and a vertex farthest from v. The *radius* of G is the minimum eccentricity of a vertex in G. We determine sharp bounds on the maximum number of edges in a bipartite graph of given order and radius.

The depression of a graph — Part 1

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SAMS Subject Classification: 14

The notions of the *flatness* of an edge–ordering and of the *depression* of a simple graph are introduced in this talk. Some general properties of these parameters are established, after which exact values for the depressions of a number of simple graph classes are found exactly and contrasted against those of a previously established parameter, called the *altitude* of a graph, which is, in a sense, the dual to the depression parameter.

A crossing number lower bound via algorithmic graph–to–graph embedding

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SAMS Subject Classification: 14

Leighton introduced the concept of graph–to–graph embedding in which a graph G with a known lower bound to its crossing number is embedded into a graph H. The "density" of the embedding into H may then be used to compute a lower bound to the crossing number of H. Given such an embedding, Leighton showed that a lower bound could be given in closed form. Shahrokhi, Székely, Sýkora and Vr'to improved Leighton's analytical bound. However, they did not considered the problem of determining such embeddings algorithmically — this is an important problem, and such an algorithmic method is discussed in this talk.

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16 Computational methods

Angular deviation of light rays that emerge from optometric prisms - near linear behaviour of nonlinear phenomena

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SAMS Subject Classification: 16,17

Incident rays emerging from glass prisms are deviated. The angles of deviation depend on the material of the prism, the apex angle and the angle of incidence. For a prism in air, the incident angle ϕ_1 is allowed to increase from its least value ϕ_{min} for a series of apex angles α. For each α-value and each ϕ_1 the corresponding deviation δ-values are calculated. As α varies, a graph of δ_{max} versus ϕ_{min} appears to be linear as does a graph of δ_{min} versus ϕ_l (the angle at which the ray emerges in air). The expressions for δ_{max} and δ_{min} are nonlinear. We fit a continuous linear least-squares approximation to δ_{max} and δ_{min}, performing the integration numerically as analytical integration is not feasible. A very good fit is obtained with correlation coefficient close to unity.